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A MATHEMATICAL MODEL FOR AIR FLOW IN A VEGETATIVE CANOPY

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1. INTRODUCTION

The usual concept of surface roughness and zero plane displacement breaks down when one considers a vegetative canopy as the lower boundary for the planetary boundary layer. The roughness of many vegetative canopies derived from the log law changes with wind speed as do the zero-plane displacements. Ideally one would like to express the aerodynamic roughness of vegetation in terms of its height, density and drag characteristics.

Restricting considerations to the turbulent transfer of momentum, a model was developed that will predict the canopy wind profile within semi-rigid canopies. For the purpose of this report, a canopy is defined as that layer spanning the region from the ground surface to the top of the plant. To date three canopy flow models have been developed. This paper primarily describes the third model. It also discusses the two previous models (Cionco, Ohmsted and Appleby, 1963) briefly, and then compares this prior work to that of other investigators.

2. BACKGROUND

The earliest efforts in canopy modelling were reported by Lemon and Stoller, Tan and Ling, and Ordway, Ritter, Spence and Tan (Lemon et al, 1963). In 1960 Ordway et al (1960) proposed a simple canopy flow model. It was suggested that the turbulent transfer in the canopy could be pursued in a manner similar to that of the surface boundary layer, provided allowance is made for the loss of momentum to the leaves and stalks of the vegetation. They further proposed that the loss of momentum was proportional to the square of the local mean velocity.

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This condition is analogous to the definition of the drag coefficient for fully rough flow through a pipe. Under this condition the drag coefficient is independent of the Reynolds number. Also, for the steady state condition with no advection, the local loss of momentum must be equal to the convergence of momentum transport. To simplify their model, Ordway et al assumed the transfer coefficient,  $K_m$ , to be constant within the canopy. In a later report, Tan and Ling (1961) allowed  $K_m$  to increase linearly with height as it does in the surface boundary layer. The author's initial canopy flow model (Cionco, 1962) is similar to these models in many respects with one important exception: the model does not assume what the behavior of  $K_m$  is within the canopy, and rather than working with the transfer coefficient, it was more desirable to investigate the mixing length and its properties. Thus for turbulent transfer of momentum within vegetative canopies the following equations result:

$$\frac{\partial \tau}{\partial z} = \rho S u^2 \quad (1)$$

or

$$\frac{\partial}{\partial z} \left[ l \frac{\partial u}{\partial z} \right]^2 = \frac{1}{2} C_D' A u^2 \quad (2)$$

where  $\tau$  is the shearing stress,  $z$  is the vertical space coordinate,  $\rho$  is the density and  $u$  is the mean wind velocity.  $S$  is defined as:

$$S = \frac{1}{2} C_D' A \quad (3)$$

and is similar to Prandtl's skin friction coefficient.  $C_D'$  is the drag coefficient of the canopy elements,  $A$  is the effective aerodynamic surface area of the vegetation per unit volume and  $l$  is the mixing length. This model yielded a mixing length that was essentially constant with height in a corn plant canopy. The wind profile solutions for a semi-rigid canopy using these general mixing characteristics in equation (2) were in excellent agreement with the observed canopy wind profiles.

The second model formulated a canopy of elements that are ideally uniform in geometry, distribution and behavior. The basic assumptions of this "ideal" canopy model are that the scale length of turbulence and the dissipation coefficient are constant with height. Thus the idealized canopy can be defined as having (1) a uniform vertical leaf area distribution, (2) a uniform vertical distribution of the drag coefficient of the canopy elements, and (3) a drag coefficient that is

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independent of the local Reynolds number. The properties of this model are characterized by (1) a mixing length that is constant with height except very near the surface, (2) the turbulence intensity is constant with height, and (3) the wind velocity is an exponential function of height of the following form:

$$U = U_H e^{a\left(\frac{z}{H} - 1\right)} \quad (4)$$

where the attenuation coefficient "a" is a constant. The model also defines relationships of the canopy's basic parameters in terms of "a" in the following forms:

$$a = \frac{HS}{C_D} \quad (5)$$

and

$$a^3 = \frac{H^3 S}{2L_0^2} \quad (6)$$

where "a" is constant if both S and  $L_0$  are unchanging with height. A third relation evolves from the above a-terms as:

$$a^2 = \frac{H^2 C_D}{2L_0^2} \quad (7)$$

It relates the drag at the canopy top to the mixing length within the canopy.

When applying the "ideal" model to the natural canopy during the steady state condition with no advection, it was found that (1) the wind profiles within a corn plant canopy reported by Tan and Ling (1961) were exponential and that the mixing length was computed to be constant with height, (2) results reported by Nakagawa (1956) for a rice crop and by Inoue, Lemon and Denmead (1964) for a wheat field have shown turbulence intensity to be almost independent of height, and (3) limber crops like alfalfa (Lemon et al, 1963) were definitely not ideal in nature. Cionco et al also found that S,  $C_D$ , and  $L_0$  were not conservative properties of the canopy, but were interdependent and only "a" as expressed in equation (5) was nearly constant. The other relations in equations (6) and (7) were sensitive to windspeed. Saito (1964) also points out that the condition "a" = constant through all heights does not seem to be satisfied in the general case.

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Other investigators working with more recent canopy data have found similar results. Inoue (1963) discussed corn, rice and wheat canopies. Inoue, Lemon, and Denmead (1964) reported a wheat canopy. Uchijima and Wright (1963) described an immature corn canopy, and Saito (1964) discussed a wheat canopy and two separate immature corn canopies.

Inoue, Saito, and Inoue et al have also found that the constant mixing length and the exponential wind equation apply to their canopy studies. Except for some of the immature crops, their attenuation coefficients were essentially constant for the respective crop canopies. Uchijima and Wright fitted a log-linear function of height and velocity for their profiles; they also described the mixing length to increase linearly with height. However, Inoue (1963) pointed out that the data does not support their conclusion for  $\lambda$ . He analyzed  $\lambda$  to be essentially constant throughout most of the canopy. One other interesting result of these analyses is that Inoue et al and Inoue also established the relation proposed by Cionco et al shown as equation (6).

### 3. MIXING LENGTH MODEL AND SOLUTIONS

The studies and models described thus far all define the resistive effect of the vegetation in the form proposed by Ordway et al (1960). They follow Prandtl's skin friction concept (Prandtl and Tietjens, 1957). However, the new model will depart from this concept to present a more realistic simulation of the natural canopy. This change is suggested by the work reported by Kutzbach (1961). Empirical results derived from his field studies indicated another form of  $S$  (defined by equation (3)) would be more appropriate for field observations. Kutzbach conducted an experiment to determine the effect of artificial roughness elements upon the wind profile over an ice covered lake. The roughness elements were ordinary bushel baskets and the experiment investigated the orderly variations of both the density and heights of the roughness elements. The data revealed that  $S$  should be redefined so that the density function of the roughness elements has more meaning. Further analysis of his observations indicated that the parameter  $A$  should be a squared term.<sup>1</sup> To keep  $S$  dimensionally correct and

<sup>1</sup>The bushel basket analysis yielded  $C_D A C_D^{0.7}$  and also  $C_D A^{1.243}$  so that  $S A^{1.8}$ . For computational convenience, 1.8 was rounded off to 2 so that  $S A^2$ . See page 25 of Cionco et al (1963) for these relationships.

consistent with the previous theoretical model (Cionco et al, 1963) an additional length term is necessary. The most obvious length parameter to be considered for the canopy flow model is the mixing length in the crop. Equation (8) is the definition of  $S'$  based on the bushel basket results.

$$S' = \frac{1}{2} C_D^1 A^2 l \quad (8)$$

Equation (2) can now be written making use of equation (8):

$$\frac{\partial}{\partial z} \left[ l \frac{\partial u}{\partial z} \right]^2 = \frac{1}{2} C_D^1 A^2 l u^2 \quad (9)$$

A solution of the above equation, of course, poses a problem in that there are more unknowns than equations. All of the components of  $S'$  are not known.  $C_D^1$  in particular is unknown.  $A$  is known and can be evaluated graphically from the agronomist's cumulative leaf area index shown in Figure 1 (Allen et al, 1964). The mixing length is also unknown and, of course, the velocity distribution is the intended solution. Of these unknowns, the only data available are wind profiles in the canopy. This fortunately is sufficient information to allow for a solution of  $l$  and  $C_D^1$  as one unknown. The intention now is to provide enough information to establish some general properties of the canopy's mixing processes and drag characteristics.

The mixing-length solution of equation (9) can be expressed as follows in a normalized form:

$$\frac{\partial}{\partial X} \left[ \frac{L}{BL^{1/2}} \right] = \frac{A^2 Y^2}{2(\partial Y / \partial X)^2} - \left[ \frac{L}{BL^{1/2}} \right] \frac{\partial^2 Y / \partial X^2}{\partial Y / \partial X} \quad (10)$$

For computational purposes it was desirable to express the solution in nondimensional terms. The parameters were arbitrarily normalized with respect to a level twice the height of the canopy. The nondimensional variables are  $X = Z/2H$ ,  $Y = u/u_{2H}$  and  $L = l^2/l_{2H}^2$ . The  $B$ -term is a collection of constants along with  $C_D^1$  and is defined as  $B = 8H^3(\frac{1}{2} C_D^1)/l_{2H}^2$ . Equation (10) is simply the result of differentiating equation (9) by parts and dividing through by  $L$ . Thus given a canopy wind profile and the vertical leaf area distribution, equation (10) will yield a solution of  $L$  and  $C_D^1$ .

The computer form of equation (10) is:

$$\frac{\partial E}{\partial X} = C_1 + EC_2 \quad (11)$$

where  $E = L B^{-1} L^{-1/2}$  and  $C_1$  and  $C_2$  are the respective righthand terms of the previous expression. Equation (11) is concerned with two distinct regions of turbulent flow—that within the canopy and that immediately above. For  $Z > H$ ,  $C_1$  must be zero for this expression to apply in the region where the logarithmic wind profile is valid.

The first mixing length solutions of equation (11) were calculated utilizing Tan and Ling's (1961) two complete wind profiles shown in Figure 2. These same complete profiles are shown in Figure 3 to depict their fit of the exponential wind profile of equation (4). Second and third order lagrangian polynomials were used to estimate the velocity derivatives of  $C_1$  and  $C_2$  from the normalized data. The initial conditions for the  $z = 0$  level are  $C_1 = \text{zero}$  and  $C_2$  defined by  $2H/z_0$ , where  $z_0$  was the roughness length at the ground surface.  $E$  as defined for equation (13) was approximately  $10^{-4}$  for  $z = 0$  and  $B$  is of the order of  $10^2$  for a semi-rigid canopy crop. The  $A$ -term was constant from the ground surface to the uppermost 20% in which the density of the foliage decreased uniformly to zero at the top of the vegetation. Equation (11) was then integrated from the ground surface to  $2H$  using a fourth-order Runge-Kutta integration routine (Nielsen, 1956).

The solutions are shown in Figure 4 as profiles of the normalized mixing-length term,  $E$ , versus the normalized height,  $X$ . The height has been normalized so that the velocities and mixing lengths would be representative of a unit canopy. Within the canopy, the mixing length is essentially constant except near the ground surface; above the canopy the mixing length increases linearly with height. The solution for Corn 1 firmly supports these conclusions, while the results for Corn 2 are less consistent within the canopy.

In a general way, these solutions are compatible with the idealized canopy concept of Cionco et al and conform to the canopy scheme proposed by Inoue. The nearly constant mixing lengths calculated from the profiles confirm the interdependence of  $S'$  and  $\lambda$  with height.

## 4. CANOPY FLOW MODEL AND SOLUTIONS

In the previous section, it was necessary to establish some general characteristics of the canopy's mixing processes. With this information now available, a solution of the canopy wind profile is possible. Equation (9) can be expressed as:

$$\frac{2}{H} \frac{\partial u}{\partial X} \left[ \frac{\partial u}{H^2 \partial X} - \frac{\partial(\ln \epsilon_0)}{\partial X} + \frac{\partial^2 u}{H^2 \partial X^2} \right] = \frac{S' u^2}{\epsilon_0^2} \quad (12)$$

for  $Z \leq H$ . All parameters except  $X$  retain the same definitions as previously. The model treats the canopy as a unit, thus  $X = Z/H$ .

The mixing length solutions also confirm Inoue's (1963) representation of the canopy mixing length by two expressions:  $\epsilon = kz$  near the earth's surface and  $\epsilon =$  constant throughout the rest of the canopy. By accepting this generalized mixing length in the canopy,  $\partial(\ln \epsilon_0)/\partial X$  can be determined. To allow for the above changes in the mixing length in the righthand term, a generalized  $\epsilon^2/\epsilon_0^2$  as a function of height was computed from the mixing length solutions. Defining two new variables of the righthand term as  $C = H^3 S'/\epsilon_0^2$  and  $G = \epsilon^2/\epsilon_0^2 A^2$ , equation (12) can now be written as:

$$2 \frac{\partial u}{\partial X} \left[ \frac{\partial u}{\partial X} - \frac{\partial(\ln \epsilon_0)}{\partial X} + \frac{\partial^2 u}{\partial X^2} \right] - \frac{C}{G} u^2 = 0 \quad (13)$$

The  $C$ -term is now the only unknown. To determine  $C$ , an empirical relationship is necessary. From the ideal canopy concept, several relationships were established as equations (5, 6, and 7) that should be unique functions of the canopy. Equation (5) evaluated from the mature corn data proved to be reasonably constant and equal to 2.2. Equation (7), however, varied as a function of wind speed. When modified to  $\beta = U_H H^2 C_D / 2 \epsilon_0^2$  it became reasonably constant and equal to 713.3. From these relationships,  $C$  can easily be determined as a function of  $U_H$  where  $C = 2\beta\beta/U_H$ .

The canopy wind profile solution is obtained from a nonlinear overrelaxation algorithm described by Lieberstein (1959). It is an iteration method that continually solves for the best estimate (EST) of the velocity such that equation (13) converges to zero; i.e.,

$$\text{New EST} - \text{Old EST} - \frac{(\text{EQ } 13)}{2(\text{EQ } 13)/\partial u} \quad (14)$$

Second order lagrangian polynomials are used to evaluate the first and second derivatives of the velocity. The initial conditions of the algorithm are the velocity,  $U_0$ , at the ground surface, the velocity,  $U_H$ , at the canopy/air interface, and a single value of  $C$ .  $U_0$  is of course zero.  $U_H$  is observed or computed from the log law relationships, and  $C$  is evaluated directly from the mixing length solutions or from the  $\alpha$  and  $\beta$  relationships. From this, one sees that a solution is easily attainable with the minimum information of  $U_H$ .

Canopy flow solutions of the original Tan and Ling data (Figure 2) are shown in Figure 5. This plot shows good agreement between the observed and simulated profiles of Corn #1 and #2. The other simulated profiles are in good agreement with the partially observed profiles of the upper third of the canopy. The largest differences between all of the observed and simulated values are 5% of that velocity range.

## 5. OTHER RESULTS AND DISCUSSION

More solutions were obtained as an independent check of the canopy wind profile model. Figures 6 and 7 summarize the model's simulations of the observations reported by Plate and Cermak (1963) and Wright (1963). Figure 6 consists of two separate sets of data from mature corn crop canopies. Five profiles are for Wright's data; the other data points represent five profiles for Tan and Ling's data. Figure 7 compares the simulations to Plate and Cermak's wind tunnel observations. These profiles were collected within and above a 4-inch artificial canopy of semi-rigid plastic strip elements. This plot compares ten previously simulated solutions (for mature corn) to ten wind tunnel profiles. Generally speaking, the computations were in good agreement with the observations.

Statistically, the agreement can be summarized as follows. Error analysis of the corn canopy solutions yielded an average absolute error of 11.7 cm/sec and a root-mean-square-error of 14.8 cm/sec. The artificial canopy solutions have an average absolute error of 12.7 cm/sec and a root-mean-square-error of 15.1 cm/sec. From this analysis and Figure 7, it is apparent that velocity predictions of 100 cm/sec or less are subject to sizeable



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error for the artificial canopy. Fortunately, this velocity range is usually restricted to the lower third of the canopy.

The under prediction of the low velocity range in the artificial canopy may be due to the bending of the plastic strips. As mentioned previously, Saito concluded that although "a" appeared constant for the corn canopy, it would not be true in the general case. Because the computed profiles were simulated for mature corn, "a" was considered constant for the plastic strips rather than a function of velocity and flexibility. This would also contribute to an under prediction of velocities deep in the canopy.

The above suggests that the attenuation coefficient may be a useful method to study various canopies because the exponential profile has been observed in quite a variety of vegetations. In particular the effects of different leaf types, densities and degrees of rigidity should be investigated. This coefficient should be related to the overall canopy density and flexibility. Sparse vegetation which can be readily penetrated by the eddies from above would tend to have small attenuation coefficients. These coefficients should increase as the canopy density increases. With changes in flexibility of the vegetation, one would expect the coefficient to increase with wind speed as the plant streamlines to the wind. More intensive investigation of "a" from wind profiles in canopies of known densities and elasticities may help to establish the aerodynamic characteristics of vegetation by simple relationships.

By supplementing the model with wind profiles above the canopy and canopy temperature and humidity observations, the model should prove useful in studying the canopy's energy transport and diffusion processes. In describing the sink and source distributions of the energy balance components, values of these canopy fluxes could be approximated. The diffusion of matter into the canopy could also be estimated using the functional relationship of eddy viscosity to the mixing length. In this light the generalized mixing length discussed previously will take on more importance.

## 6. SUMMARY

Although the model has limitations in that it has been applied only to semi-rigid vegetation, it does yield reasonable results for canopies that meet the model's requirements. The computed mixing length

solutions showed that  $\lambda$  was nearly constant with height in the canopy. This agreed closely to the  $\lambda$  properties proposed previously by the idealized canopy concept and later by other investigators. These solutions also showed that  $\lambda$  increased linearly with height above the canopy as it should in the logarithmic wind profile domain. Finally, the computed canopy flow solutions were in good agreement with the observations. These solutions further verified the concept that the canopy's mixing length profile is essentially constant.

## 7. ACKNOWLEDGMENTS

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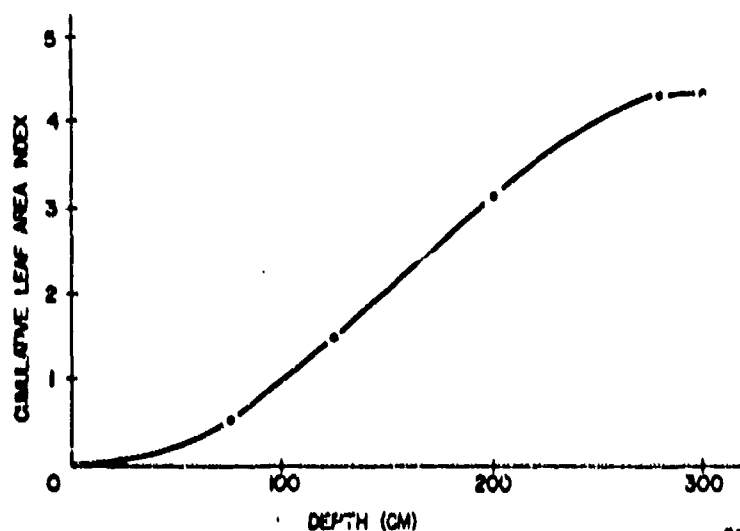


Figure 1. Cumulative Leaf Area as a Function of Height in a Mature Corn Canopy (Allen, et al, 1964)

Figure 2. Wind Profiles Above and Within a Mature Corn Canopy (Tan and Ling, 1961)

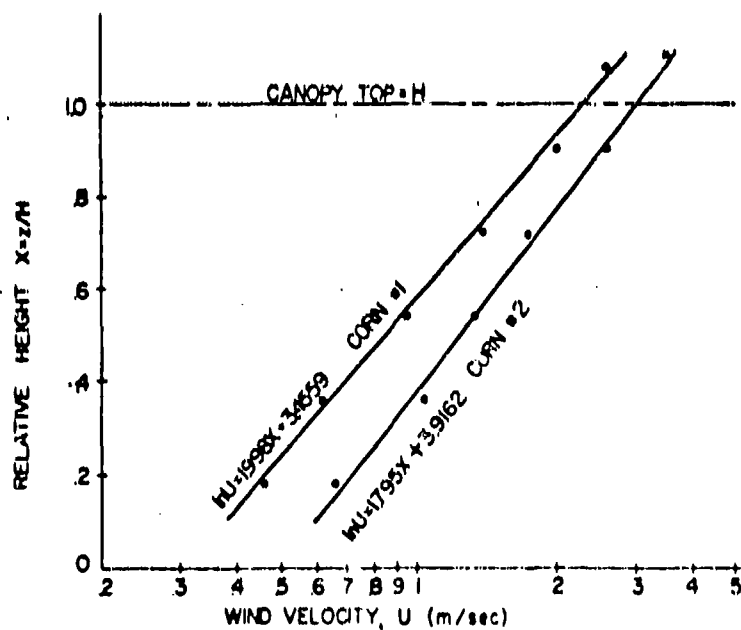
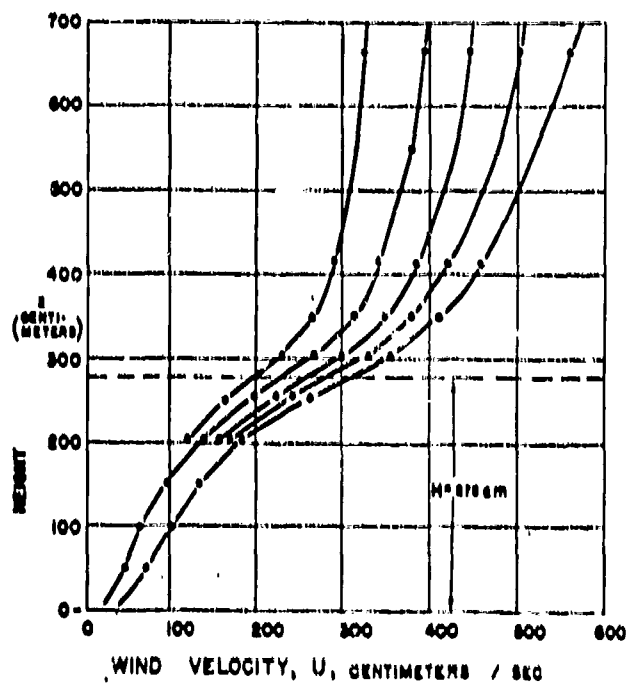


Figure 3. The Logarithm of the Wind Speed as a Function of the Relative Height Within a Mature Corn Canopy.

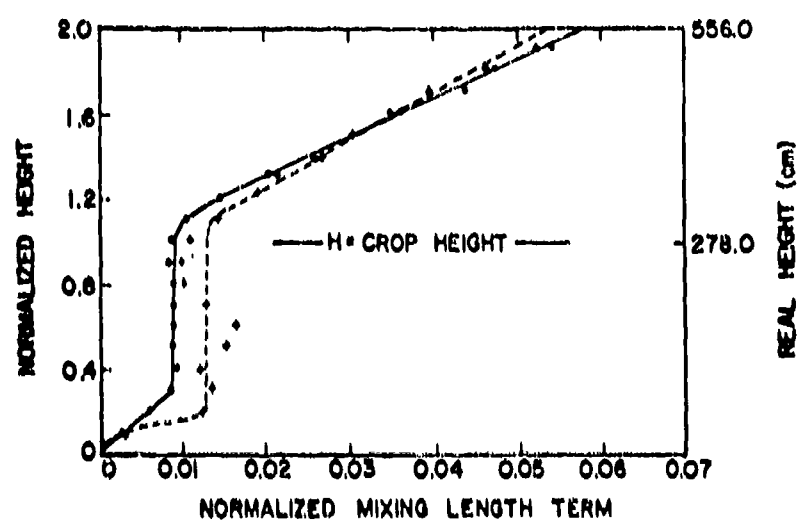


Figure 4. The Normalized Mixing Length Terms as a Function of Normalized Height Within and Above a Mature Corn Canopy

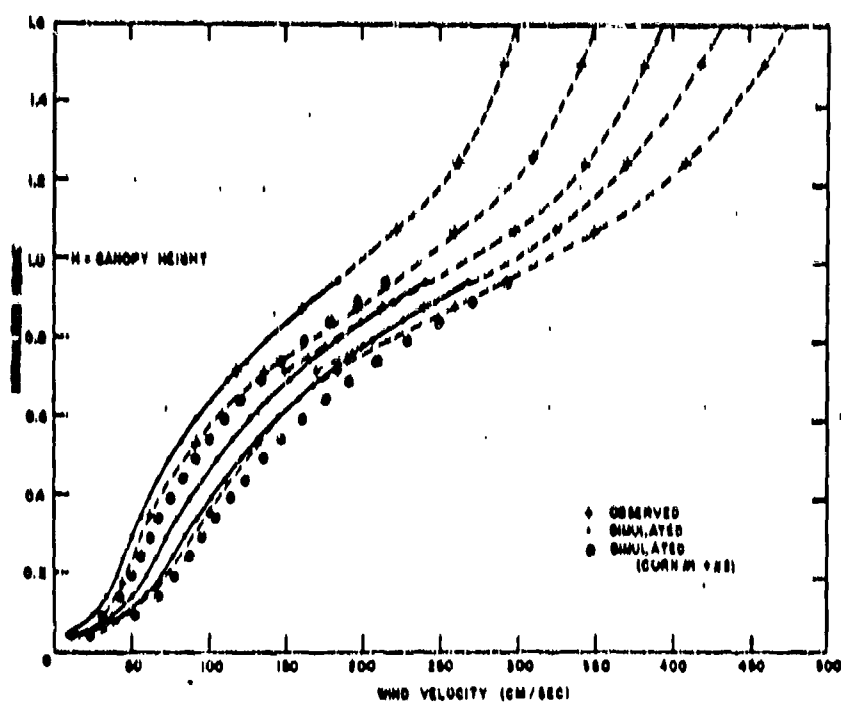


Figure 5. A Comparison of the Observed Wind Velocities both Within and Above the Canopy to the Calculations Within a Mature Corn Canopy

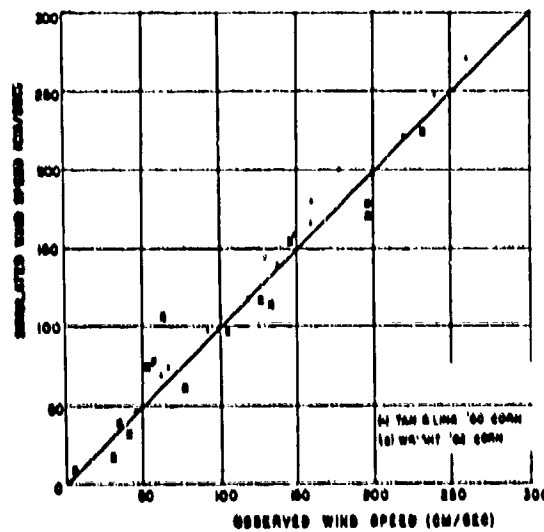


Figure 6. A Simple Scatter Diagram Comparing Calculated and Observed Velocities for a Mature Corn Canopy. The curve is the line of perfect prediction and does not represent the data.

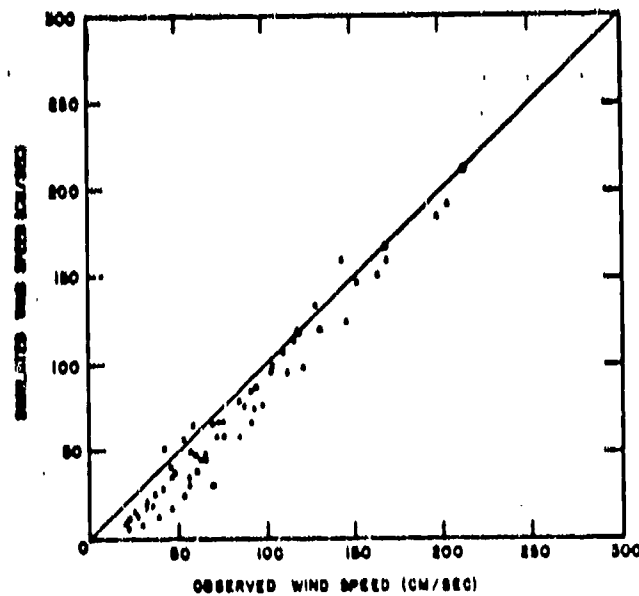


Figure 7. A Simple Scatter Diagram Comparing Calculated and Observed Velocities for an Artificial Canopy. The curve is the line of perfect prediction and does not represent the data.